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Isometrical scaling between leaf volume and mass

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Key words: allometric law, leaf, metabolic rate

1 SUMMARY

Different leaf samples were randomly chosen, and the mass and the volume of each sample were measured immediately to determine the allometry of leaf volume and fresh mass. In this study, instead of dry mass, fresh mass is used because water is an essential element for life. The results show an isometrical scaling between leaf volume and fresh mass.

2 INTRODUCTION

Nature endows life an additional dimension (West et al, 1999; He, 2009). The standard allometric approach to a biological process is to seek a relationship between body size and metabolic rate across all organisms (West et *al*, 1997). This might be the main pitfall of the previous theories. Actually different organs in mammals have different allometric cascades (Wang, O'Connort, Heshka & Heymsfield, 2001)and this should be also valid for plant study. Reich et al. found that the whole-plant respiration rate isometrically scales with total plate mass (Reich, Tjoelker, Machado, et al., 2006), refuting the early confirmation of 3/4 scaling law (West. Brown, Enquist, 1997). Our observation on the allometry of leaf volume and fresh mass elucidates another new refutation of the 3/4 scaling law in plants where an explanation is given using life-dimension (West. Brown, Enquist, 1999; He, 2009).

3 ALLOMETRIC LAW

The fourth dimension of life (West et al, 1999; He, 2009) represented by the Kleiber 3/4 allometric law enables us to estimate the mass of an animal with ease (West et al,1997; 1999; He, 2009)

$$H \propto M^{1/4} \quad (1)$$

where H is the height of an animal, M the body mass.

The simplest way to estimate the weight of an animal, however, might be to calculate its volume. By simple geometric analysis, we have

$$V \propto H^3 \quad (2)$$

where V is the estimated volume. The volume is proportional to body mass, M, so we have

$$M \propto H^3 \quad (3)$$

The derivation of Equation. (3) is absolutely wrong (He 2009; He, Yang & Mo, 2009). If Equation. (3) was valid, life and nonlife would not be so different (He 2009; He, Yang & Mo, 2009). Actually nature endows life an additional dimension (West et al. 1999; He, 2009; He & Huang, 2006), instead of Equation. (3), we have

$$M \propto H^{3+1} \quad (4)$$

which is exactly same with Equation.(2).

Enquist and Kiklas (2002) suggested a general allometric model for plant biomass: total leaf area scales $M_T^{3/4}$, where M_T is the total plant dry mass, however, the 3/4 scaling law for plants has been refuted by many authors. Sack et al. found experimentally another scaling law for early plant ontogeny . The specific leaf area scales $M_T^{-0.22}$ (Sack et al, 2002), the explanation of Sack et al.'s experiment was given by He (2005); Reich et al. found that the whole-plant respiration rate isometrically scales with total plate mass (Reich*et al.,* 2006). All observation of plants refuted the early confirmation of 3/4 scaling law.

Different organs in a living body obey different scaling laws, and the scaling exponent depends upon the cell structure and cell distribution in the organ (He, 2008). As it is well-known that four metabolically active organs, brain, liver, kidneys and heart together account for ~60% of energy expenditure in humans, even though the 4 organs represent <6% of body mass (Wang et al, 2001) , this should be also valid for plant study (He, 2005; Zhou,Jiang and Mo, 2009).

4 SCALING RELATIONSHIP BETWEEN LEAF VOLUME AND MASS

Now consider the relationship between leaf mass M and leaf volume *V*. Leaf volume is a geometrical parameter, so it follows

$$V \propto r^3 \tag{5}$$

where r is a length unit on cell's scale; while the leaf mass is a bio-related parameter. The simplest way to estimate M might be to calculate its volume, this leads to the result:

$$M \propto \delta S \propto S \propto r^2 \tag{6}$$

where δ is the thickness of the leaf, which is a constant for most leaves.

As mentioned above, the derivation of Equation.(6) is, however, NOT true. Endowing an additional dimension, Equation (6) is changed as:

$$M \propto r^{2+1} \tag{7}$$

From (5) and (7), we can immediately obtain the following scaling relationship:

$$V \propto M \tag{8}$$

In order to verify our prediction, various leaf laminas were picked at the Yan-an Xilu campus of Donghua University at 6:30, July 4, 2009, and their mass measured using an electronic balance immediately. Then each leaf lamina was broken to pieces, and put the broken pieces put into a small measuring cup to measure its volume. The sample photos are illustrated in Figure 1, and the experimental data are given in Table 1.

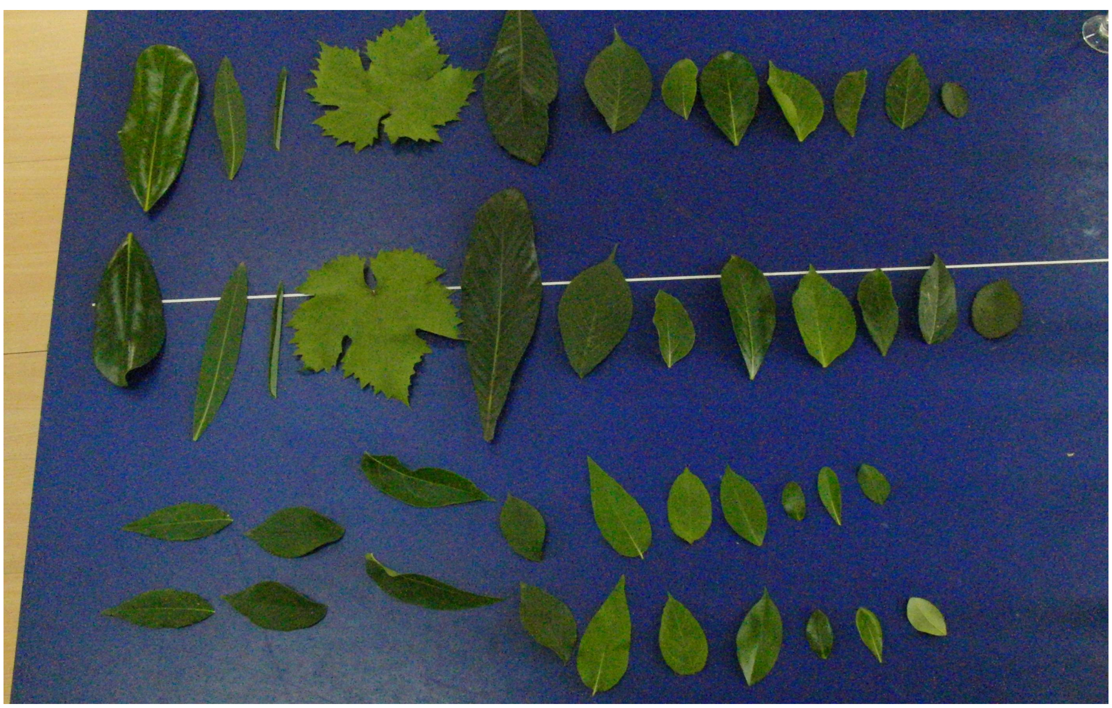

Figure 1: Samples used in the experiment

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Table 1: Volume and fresh mass of various leaves

V/ml	0.2	0.27	0.4	0.44	0.5	0.51	0.7	0.78	0.91	0.95	0.98	
M/mg	0.2	0.19	0.23	0.32	0.31	0.46	0.42	0.65	0.49	0.59	0.58	
V/ml	1.1	1.1	1.1	1.6	2.1	2.2	2.2	2.9	3.9	4.8	6.6	28.2
M/mg	0.79	0.79	0.74	0.84	1.52	1.53	1.86	1.88	2.98	4.55	5.92	21.33

Using least squares regression method, the following isometrical scaling was obtained approximately (see Figure 2):

$$M = 0.766V \tag{9}$$

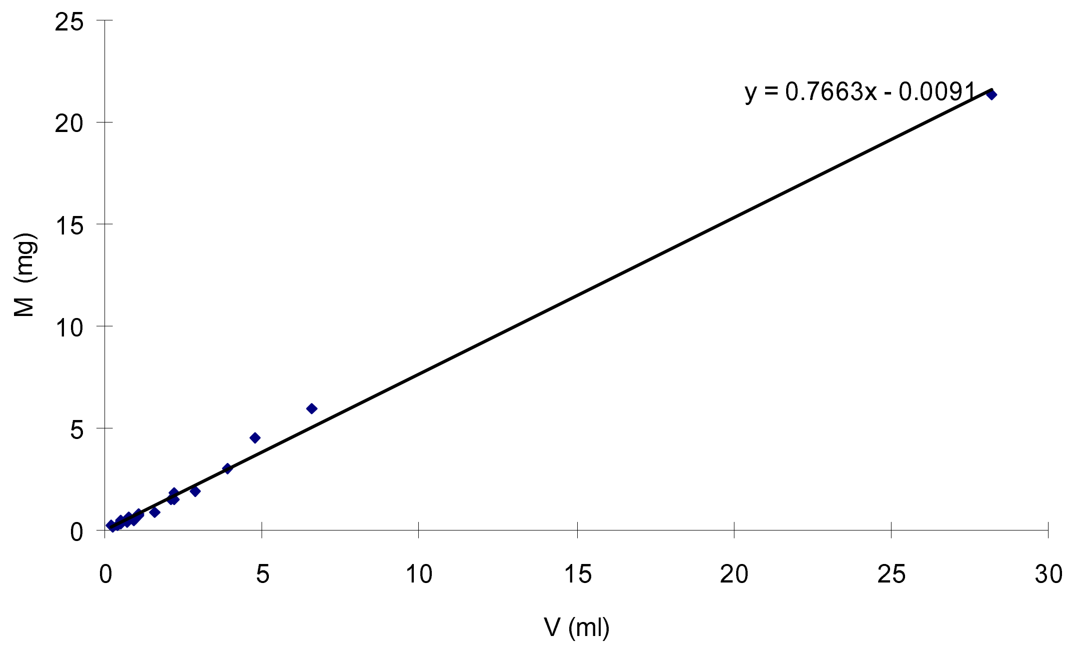

Figure 2: Relationship between the leaf volume and fresh mass

Considering $V \propto \delta S \propto S$, we can obtain also an isometrical scaling between leaf surface and fresh mass

$$S \propto M \tag{10}$$

Recently Niklas et al.'s observation (Niklas, Cobb & Spatz, 2009) on the allometry of leaf surface area and dry mass shows

$$S \propto M_t^{\alpha<1.0} \tag{11}$$

where M_t the dry mass, and scaling exponent is α is generally significantly less than one(Niklas et al, 2009). This error arises in the dry measure. Water is an essential element for life, so the mass used in our prediction must include water mass, i.e., fresh mass must be used in our prediction.

5 DISCUSSION

The study obtained an isometrical scaling between leaf volumes and mass. The previous prediction using dry mass must lead to wrong results because water is an essential element for life. The studies theoretical prediction agrees well with the experimental data, showing reliability of the isometrical scaling law, and rebutting the well-known 3/4 scaling law. The obtained isometrical scaling law can explain Reich et al.'s findings that the whole-plant respiration rate isometrically scales with total plate mass (Reich et al, 2006).

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